Int. J. Heat Mass Transfer. Vol. 14, pp. 263–278. Pergamon Press 1971. Printed in Great Britain

A MULTIPLE-BAND FORMULATION FOR RADIATIVE TRANSFER IN A SLIGHTLY DISTURBED GAS, WITH APPLICATION TO RADIATIVELY DRIVEN ACOUSTIC WAVES

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(*Received* 17 *February* 1970 *and in revised form* 15 *May* 1970)

Abstract—A linearized, purely differential equation for the radiative heat flux is obtained for a nongrey gas that is in local molecular equilibrium and slightly disturbed from radiative equilibrium. This equation is derived for a plane-parallel geometry by assuming that the spectral absorption coefficient can be represented by an arbitrary number of grey bands and by using the substitute-kernel (exponential) approximation. The order of the equation is $2N$, where N is the number of grey bands. The formulation of boundary conditions for use with the equation is also discussed. The equation is applicable in one-dimensional radiative-transfer problems whenever small disturbances from a uniform reference state are being considered.

The utility of the equation is demonstrated by using it to solve a problem of radiative acoustics—specifically, that of radiatively driven, harmonic acoustic waves in a gas between two walls. Analytical two-grey-band solutions are obtained for the pressure response at the nondriving wall in what, practically speaking, is a low- or moderate-temperature approximation. These two grey-band solutions are such that they can be extended first to multiple grey bands by suitable summations, and finally to a continuous spectrum by a transition to integrals over spectral frequency. This final extension, in effect, removes the original band-model approximation in this particular acoustic problem.

NOMENCLATURE

A_S, A_T,	isentropic and isothermal wave operators defined in equation (11);
$a, b, a_j, b_{j,k}, a_\nu, b_\nu, a_{1,2}, b_{1,2}$,	exponential-approximation constants;
a_{S0},	isentropic sound speed;
B_ν,	monochromatic Planck function;
B_j,	Planck function integrated over a limited frequency range, see equation (3);
Bo,	Boltzmann number defined in equation (25);
$Bo_j, Bo_{1,2}$,	Boltzmann numbers for grey bands defined as in equation (11);
Bu,	grey-gas Bouguer number defined on Fig. 1;
$Bu_{j,k}, Bu_{1,2}$,	grey-band Bouguer numbers defined as in equation (11);
Bu_ν,	spectral Bouguer number defined in equation (25);
C_m, c, c_m, c_{1-6},	constants used in acoustic-wave solution;
$E_n(z)$,	exponential-integral function with argument z;
h,	specific enthalpy;

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L,	x location of right-hand wall;
l,	direction cosine of direction of radiative propagation;
N,	number of grey bands;
p,	pressure;
$Q^R_{\nu\pm}$,	one-sided, monochromatic radiant heat fluxes;
q^R_ν,	net monochromatic radiant heat flux;
q^R,	net radiant heat flux;
q^R_j,	net radiant heat flux in band j;
R,	gas constant;
T,	temperature;
T_A,	dimensionless amplitude of wall-temperature perturbation, see equation (18);
T_w,	wall temperature;
t,	time;
u,	velocity;
x,	space coordinate;
α,	grey-gas absorption coefficient;
$\alpha_{j,k}, \alpha_{1,2}$,	grey-band absorption coefficients;
α_ν,	monochromatic absorption coefficient;
γ,	ratio of specific heats;
ν,	spectral frequency;
$\Delta\nu_j$,	spectral extent of a grey band;
ρ,	density;
σ,	Stefan–Boltzmann constant;
φ,	velocity potential;
ω,	acoustic frequency.

Subscript

0,	evaluated at the reference condition.

Superscripts

$'$,	perturbation quantity;
$\bar{}$,	normalized quantity;
$\tilde{}$,	dummy variable of integration.

1. INTRODUCTION

THE GREY-GAS assumption in radiative transfer has been popular because it shows trends and in some cases leads to qualitatively correct answers. However, in any comparison with experiment or other attempt to provide quantitatively accurate calculations for a real gas, results on the basis of the grey-gas approximation are not trustworthy. It is therefore desirable to have formulations that are independent of this restriction. One such formulation, applicable in linearized transfer problems, has been given by Gilles *et al.* [1], who used a modified (nongrey) substitute-kernel approximation to simplify, essentially simultaneously, the spectral and angular integrations that appear in the radiative heat-flux equation. The simplicity of such an approach is appealing; however, evaluating the constants in the substitute kernel may be tedious, and the accuracy of the approximation is open to question. We present here an alternative formulation, also for linearized problems, based on the well-known idea of approximating the spectral variation of the absorption coefficient by means of grey bands, i.e. by constant values within nonoverlapping spectral regions. In addition, we restrict the problem to one spatial dimension and use the exponential approximation to the exponential-integral functions. This band-model approach is similar to that employed by Liu and Clarke [2] and Olfe and Cavalleri [3], who used two grey bands in conjunction with the differential approximation for radiative transfer.

We begin in Section 2 by formulating a purely differential equation [equation (8)] for the perturbation in radiative heat flux when radiative transfer occurs in an arbitrary number of grey bands. The order of this equation depends on the number of bands and increases by two for each additional band. We also discuss the formulation of boundary conditions for use with this equation. To show the utility of the grey-band model, we use it in Section 3 to formulate the problem of radiatively driven, harmonic acoustic waves in a gas confined between two parallel walls (cf. Long and Vincenti [4]). We then solve this problem in Section 4 for two grey bands with the approximation, valid at low or moderate temperatures, that a certain grouping of dimensionless parameters is small and thereby obtain analytical results for the pressure response at one of the walls. Finally, we generalize these results to apply to a continuous spectrum by obtaining the pressure response in terms of integrals over spectral frequency. This final step, in effect, removes the original grey-band assumption from the acoustic results. This allows comparison with the solutions from Cogley and Compton [5], who solved essentially the same problem but with a somewhat different approximation. We will show that we can by the present approximation obtain results for one situation in which the approximation used by Cogley and Compton fails.

The differential equation for radiative heat flux that will be derived [equation (8)] is not limited to problems of radiative acoustics, but is applicable to any radiative-transfer problem in a plane-parallel geometry as long as the variations in thermodynamic properties are small enough that a linearized treatment is valid.

2. FORMULATION OF RADIATIVE HEAT-FLUX EQUATION

The geometrical arrangement consists of a radiating gas confined between two plane-parallel walls. The left-hand wall located at $x = 0$ is a black body, while the right-hand wall at $x = L$ is a perfect reflector. For this one-dimensional geometry the one-sided perturbation heat fluxes (right- and left-directed respectively) can be written as (cf. [1])

$$Q^{R'}_{\nu+} = 2\pi \frac{\mathrm{d}B_\nu}{\mathrm{d}T}\bigg|_0 \left\{ T'_w E_3[\alpha_{\nu 0} x] + \int_0^x \alpha_{\nu 0} T' E_2[\alpha_{\nu 0}(x - \tilde{x})]\,\mathrm{d}\tilde{x} \right\},$$

$$Q^{R'}_{\nu-} = 2\pi \frac{\mathrm{d}B_\nu}{\mathrm{d}T}\bigg|_0 \left\{ T'_w E_3[\alpha_{\nu 0}(2L - x)] + \int_0^L \alpha_{\nu 0} T' E_2[\alpha_{\nu 0}(2L - x - \tilde{x})]\,\mathrm{d}\tilde{x} \right.$$

$$\left. + \int_x^L \alpha_{\nu 0} T' E_2[\alpha_{\nu 0}(\tilde{x} - x)]\,\mathrm{d}\tilde{x} \right\} \qquad (1)$$

The subscript 0 denotes the undisturbed reference condition, for which radiative equilibrium prevails, and primed quantities are small perturbations from that condition. For example, the temperature is given by $T = T_0 + T'$. The quantities α_ν and B_ν are the monochromatic volumetric absorption coefficient and Planck function respectively. The perturbations in temperature of the gas and the left-hand wall are given by T' and T'_w. The quantity E_n is the exponential-integral function of order n defined by

$$E_n(z) \equiv \int_0^1 \exp(-z/l) l^{n-2}\, \mathrm{d}l.$$

The net monochromatic heat flux $q_\nu^{R'}$ is obtained by combining equations (1) according to $q_\nu^{R'} \equiv Q_{\nu+}^{R'} - Q_{\nu-}^{R'}$.

In order to perform the spectral integration of equations (1), we assume that the spectral variation of the absorption coefficient is given by a series of non-overlapping grey bands, i.e. $\alpha_{\nu_0} = \alpha_{j_0}$ within $\Delta\nu_j$. Obviously, by choosing many bands we can in principle approximate any actual spectral variation. Also, as is often done in one-dimensional radiative transport, we assume that the exponential-integral functions can be satisfactorily approximated as pure exponentials. Here we use a separate exponential approximation for each of the grey bands, i.e.

$$E_n(z_j) \approx a_j b_j^{2-n} \exp(-b_j z_j).$$

We make this multiple exponential approximation because values for a and b that give good accuracy for a grey gas or for a single grey band can sometimes be found either from comparison between exact and approximate solutions (e.g. [5]) or by matching certain properties of the exact and approximate E functions in limiting cases (e.g. [6]). These values for a and b that provide good precision usually depend on the optical thickness of the gas and hence on the value of the grey-gas or single-grey-band absorption coefficient. Thus, in an application of the present technique, the user can if he wishes choose values for a_j and b_j that are based on the individual optical thicknesses or absorption coefficients of his multiple grey bands.

With the multiple-grey-band assumption and the exponential approximation we can write the net flux $q_j^{R'}$ in each band as

$$q_j^{R'} = 2\pi a_j \left.\frac{\mathrm{d}B_j}{\mathrm{d}T}\right|_0 \left\{\frac{T'_w}{b_j}\{\exp[-b_j\alpha_{j_0}x] - \exp[-b_j\alpha_{j_0}(2L-x)]\} + \int_0^x \alpha_{j_0} T' \exp[-b_j\alpha_{j_0}(x-\tilde{x})]\, \mathrm{d}\tilde{x} \right.$$
$$\left. - \int_0^L \alpha_{j_0} T' \exp[-b_j\alpha_{j_0}(2L-x-\tilde{x})]\, \mathrm{d}\tilde{x} - \int_x^L \alpha_{j_0} T' \exp[-b_j\alpha_{j_0}(\tilde{x}-x)]\, \mathrm{d}\tilde{x}\right\}, \qquad (2)$$

where

$$\left.\frac{\mathrm{d}B_j}{\mathrm{d}T}\right|_0 \equiv \int_{\Delta\nu_j} \left.\frac{\mathrm{d}B_\nu}{\mathrm{d}T}\right|_0 \mathrm{d}\nu. \qquad (3)$$

The total flux $q^{R'}$ that interacts with the gas is obtained as in [1] by integrating $q_\nu^{R'}$ over those spectral frequencies for which the absorption coefficient α_{ν_0} is nonzero and is given by

$$q^{R'} = \int_{\alpha_{\nu_0} \neq 0} q_\nu^{R'}\, \mathrm{d}\nu = \sum_{j=1}^{N} q_j^{R'}, \tag{4}$$

where N is the number of grey bands.

As in the grey-gas situation, a purely differential equation for $q_j^{R'}$ can be obtained by differentiating equation (2) twice with respect to x and eliminating the common terms between the resulting equation and equation (2) to give

$$\frac{\partial^2 q_j^{R'}}{\partial x^2} - 4\pi a_j \alpha_{j0} \left.\frac{\mathrm{d}B_j}{\mathrm{d}T}\right|_0 \frac{\partial T'}{\partial x} - b_j^2 \alpha_{j0}^2 q_j^{R'} = 0. \tag{5}$$

In a similar manner, a radiative boundary condition at $x = 0$ for each band can be obtained by eliminating terms between equation (2) and its first x derivative, both evaluated at $x = 0$. We thus obtain

$$\left[\frac{\partial q_j^{R'}}{\partial x} - b_j \alpha_{j0} q_j^{R'}\right]_{x=0} = -4\pi a_j \alpha_{j0} \left.\frac{\mathrm{d}B_j}{\mathrm{d}T}\right|_0 (T'_w - T'_{x=0}). \tag{6}$$

This boundary condition expresses the energy balance for an element of gas at $x = 0$ for radiative transfer in band j.

A radiative boundary condition at $x = L$ for each band can also be written by noting that there, owing to the presence of the perfectly reflecting wall, the net flux must be zero, that is,

$$[q_j^{R'}]_{x=L} = 0. \tag{7}$$

The sets of equations represented by equations (5) and (6) are coupled through the temperature. Their solution must therefore be performed simultaneously.

In some problems (particularly those of radiative acoustics), it is often convenient to use a single differential equation that governs the total radiative flux. Such an equation can be obtained by eliminating the quantities $q_j^{R'}$ between equation (5) and its even x derivatives in favor of the total flux $q^{R'}$ and its x derivatives as obtained from equation (4). A systematic procedure for accomplishing this same end is to combine equations (2) and (4) and differentiate the resulting equation $2N$ times with respect to x. We then eliminate from the evenly differentiated equations (including the zeroth derivative) the integral terms, the terms containing the wall temperature, and the terms containing the gas temperature evaluated at the wall. Either procedure yields

$$\prod_{j=1}^{N} \left(\frac{\partial^2}{\partial x^2} - b_j^2 \alpha_{j0}^2\right) q^{R'} - \frac{\partial}{\partial x} \sum_{j=1}^{N} \left\{4\pi a_j \alpha_{j0} \left.\frac{\mathrm{d}B_j}{\mathrm{d}T}\right|_0 \prod_{\substack{k=1 \\ k \neq j}}^{N} \left(\frac{\partial^2}{\partial x^2} - b_k^2 \alpha_{k0}^2\right) T'\right\} = 0, N > 1. \tag{8}$$

This is a $2N$-order differential equation for $q^{R'}$. It can easily be combined with the gas-dynamic small-perturbation equations as we show in the next section. The restriction $N > 1$ is imposed here merely to allow us to write the equation in a compact form. When expanded for a given value of N, equation (8) contains all the equations for smaller values of N, including $N = 1$. These equations can easily be recovered by setting as many of the α_j's (or $\Delta\nu_j$'s) as required to zero and then integrating twice with respect to x for each band that is dropped.

The radiative boundary conditions at $x = 0$ for equation (8) can be obtained in differential form by a similar process. We again combine equations (2) and (4) and differentiate the resulting equation

$2N - 1$ times with respect to x. We next specialize these equations to $x = 0$ and choose from them N sets of equations with each set containing $N + 1$ equations. We then eliminate the integral terms from within each of the N sets of equations to obtain N boundary conditions. Because the N sets of $N + 1$ equations can be chosen in more than one way, the boundary conditions are not unique. All the sets of boundary conditions that can be derived are nevertheless equivalent. We have not discovered any compact way in which to write these conditions and, since they are lengthy, will not include them here.

At the reflecting wall the situation is less complicated. There the net heat flux $q^{R'}$ is zero and its first $N - 1$ even x derivatives, written in terms of the even x derivatives of equation (2) and specialized to $x = L$, yield the remaining required boundary conditions. This simplification depends, of course, on our present choice of a wall that reflects perfectly. If, for example, the wall were black, or only partially reflecting, we would obtain other boundary conditions there.

With the differential equation (8) and its boundary conditions, we have, in principle, a closed set of equations from which, given the perturbations in temperature of the gas and wall, we can compute the perturbation in radiative heat flux. Note, however, that the differential equation itself does not depend on the presence of the two walls, and holds in the absence of either or both.

3. APPLICATION TO RADIATIVELY DRIVEN HARMONIC ACOUSTIC WAVES

In this section we formulate a radiative-acoustic problem using the grey-band model of the previous section. The problem chosen is that of one-dimensional, radiatively driven, harmonic acoustic waves. This problem has been previously solved on the basis of the differential approximation by Long and Vincenti [4], who used a digital computer to obtain numerical results. A similar problem has also been solved by Cogley and Compton [5], who included a term in the equations of motion to account for the presence of viscous damping at the sidewall of an enclosing cylindrical tube. In addition, they approximated the right-hand sides of equations (1) by neglecting the integral terms and by virtue of that approximation were able to obtain analytical results independent of both the grey-gas and exponential approximations. Here we proceed analytically and will obtain results for comparison with both of the previous investigations.

The small-perturbation equations for one-dimensional time-dependent flow of a thermally perfect, radiating gas can be written [7] as

Mass: $$\frac{\partial \rho'}{\partial t} + \rho_0 \frac{\partial u'}{\partial x} = 0, \tag{9a}$$

Momentum: $$\rho_0 \frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} = 0, \tag{9b}$$

Energy: $$\rho_0 \frac{\partial h'}{\partial t} - \frac{\partial p'}{\partial t} = - \int_{\alpha_{\nu_0} \neq 0} \frac{\partial q_\nu^{R'}}{\partial x} \mathrm{d}\nu, \tag{9c}$$

State: $$\left\{ \begin{aligned} h' &= \frac{\gamma_0}{\gamma_0 - 1}\left(\frac{p'}{\rho_0} - \frac{p_0}{\rho_0^2}\rho'\right), && \text{(9d)} \\ T' &= \frac{1}{R}\left(\frac{p'}{\rho_0} - \frac{p_0}{\rho_0^2}\rho'\right), && \text{(9e)} \end{aligned} \right.$$

where t is the time, u', h', p' and ρ' are the perturbations in velocity, specific enthalpy, pressure and density, respectively, and γ is the ratio of specific heats. We define a velocity potential φ such that $u' \equiv \partial\varphi/\partial x$ and $p' \equiv -\rho_0 \partial\varphi/\partial t$, which identically satisfies the momentum equation (9b). We next eliminate the thermodynamic variables from equations (9) in favor of the potential to obtain

$$\frac{1}{a_{S_0}^2}\frac{\partial^2\varphi}{\partial t^2} - \frac{\partial^2\varphi}{\partial x^2} = \frac{\gamma_0 - 1}{\rho_0 a_{S_0}^2}\int\limits_{\alpha_{\nu_0}\neq 0} \frac{\partial q_\nu^{R'}}{\partial x}\,d\nu, \tag{10}$$

where a_{S_0} is the isentropic speed of sound given by $a_{S_0} = (\gamma_0 p_0/\rho_0)^{\frac{1}{2}}$.

We now define the following dimensionless variables, parameters, and operators:

$$\left.\begin{aligned} &\bar{x} \equiv \frac{\omega}{a_{S_0}}x, \qquad \bar{t} \equiv \omega t, \qquad \bar{\varphi} \equiv \frac{\omega}{a_{S_0}^2}\varphi, \\ &Bo_j \equiv \frac{4\rho_0 a_{S_0}^3}{\pi T_0(\gamma_0 - 1)\,dB_j/dT|_0}, \qquad Bu_j \equiv \frac{\alpha_{j0}a_{S_0}}{\omega}, \\ &A_S \equiv \frac{\partial^2\bar{\varphi}}{\partial\bar{t}^2} - \frac{\partial^2\bar{\varphi}}{\partial\bar{x}^2}, \qquad A_T \equiv \gamma_0\frac{\partial^2\bar{\varphi}}{\partial\bar{t}^2} - \frac{\partial^2\bar{\varphi}}{\partial\bar{x}^2}. \end{aligned}\right\} \tag{11}$$

The quantities Bo_j and Bu_j are the Boltzmann and Bouguer numbers [7] written here for each of the spectral bands, ω is the radian frequency of the acoustic waves, and A_S and A_T are the isentropic and isothermal wave operators. The isentropic operator A_S is the normalized left-hand side of equation (10) and is related directly to the radiative heat flux through that equation; the isothermal operator A_T is related to the perturbation temperature by $A_T = -\partial\bar{T}/\partial\bar{t}$ where $\bar{T} \equiv T'/T_0$. These relations and the normalizations (11) allow us to write equation (8) as an equation for the normalized velocity potential. In terms of A_S and A_T equation (8) becomes

$$\frac{\partial}{\partial\bar{t}}\prod_{j=1}^{N}\left(\frac{\partial^2}{\partial\bar{x}^2} - b_j^2 Bu_j^2\right)A_S = -\frac{\partial^2}{\partial\bar{x}^2}\sum_{j=1}^{N}\left\{\frac{16a_j Bu_j}{Bo_j}\prod_{\substack{k=1\\k\neq j}}^{N}\left(\frac{\partial^2}{\partial\bar{x}^2} - b_k^2 Bu_k^2\right)A_T\right\}, N > 1. \tag{12}$$

Equation (12) is a linear partial differential equation of order $2N + 3$ that governs acoustic waves in the presence of radiative transfer occurring in N grey bands. As before, the restriction $N > 1$ is imposed only to allow us to write the equation in this compact form. When expanded and specialized to a grey gas, equation (12) reduces properly to the grey-gas equation [7].

We are considering harmonic acoustic waves here. We therefore let

$$\bar{\varphi} = \sum_{m=1}^{2N} C_m \exp(c_m\bar{x} + i\bar{t}), \tag{13}$$

and substitute this expression into equation (12) to obtain the characteristic equation for the $2N$ complex quantities c_m. (The quantities C_m are found by application of the appropriate boundary conditions, which can also be written in terms of φ.) This procedure gives c_m as the roots of the algebraic equation

$$i(-1-c^2)\prod_{j=1}^{N}(c^2-b_j^2Bu_j^2) = -(-\gamma_0-c^2)c^2\sum_{j=1}^{N}\left\{\frac{16a_jBu_j}{Bo_j}\prod_{\substack{k=1\\k\neq j}}^{N}(c^2-b_k^2Bu_k^2)\right\}, N>1. \quad (14)$$

This is as far as we will carry the analysis in so general a form. Equation (14) could, of course, be solved by means of digital computer for arbitrary values of the parameters. We present instead an approximate solution and further restrict the analysis by using only two grey bands. This latter restriction will eventually be removed by inspecting the form of the solution.

4. TWO GREY-BAND SOLUTION

For two grey bands the differential equation (12) becomes

$$\frac{\partial}{\partial \bar{t}}\left(\frac{\partial^2}{\partial \bar{x}^2}-b_1^2Bu_1^2\right)\left(\frac{\partial^2}{\partial \bar{x}^2}-b_2^2Bu_2^2\right)A_S = -\frac{\partial^2}{\partial \bar{x}^2}\left[\frac{16a_1Bu_1}{Bo_1}\left(\frac{\partial^2}{\partial \bar{x}^2}-b_2^2Bu_2^2\right)+\frac{16a_2Bu_2}{Bo_2}\left(\frac{\partial^2}{\partial \bar{x}^2}-b_1^2Bu_1^2\right)\right]A_T, \quad (15)$$

and the characteristic equation (14) reduces to

$$(c^2-b_1^2Bu_1^2)(c^2-b_2^2Bu_2^2)i(-1-c^2) = -\left[\frac{16a_1Bu_1}{Bo_1}(c^2-b_2^2Bu_2^2)+\frac{16a_2Bu_2}{Bo_2}(c^2-b_1^2Bu_1^2)\right]c^2[-\gamma_0-c^2]. \quad (16)$$

Inspection of the characteristic equation suggests that the analysis can be simplified considerably if we assume both $16a_1Bu_1/Bo_1$ and $16a_2Bu_2/Bo_2 \ll 1$. For the moment we regard this as merely a mathematically convenient assumption; we will show later that it has physical utility as well. We will describe it for simplicity as the approximation $16aBu/Bo \ll 1$. With this approximation we can easily obtain the roots c_m of the characteristic equation as expansions in powers of $16aBu/Bo$. To order $16aBu/Bo$ we thus obtain

$$c_{1,2} = \pm\left(i+\frac{\left(\frac{\gamma_0-1}{2}\right)\frac{16a_1Bu_1}{Bo_1}}{1+b_1^2Bu_1^2}+\frac{\left(\frac{\gamma_0-1}{2}\right)\frac{16a_2Bu_2}{Bo_2}}{1+b_2^2Bu_2^2}\right),$$

$$c_{3,4} = \pm b_1Bu_1\left(i\frac{16a_1Bu_1(\gamma_0+b_1^2Bu_1^2)}{Bo_1.\ (1+b_1^2Bu_1^2)}+1\right),$$

$$c_{5,6} = \pm b_2Bu_2\left(i\frac{16a_2Bu_2(\gamma_0+b_2^2Bu_2^2)}{Bo_2\ \ (1+b_2^2Bu_2^2)}+1\right). \quad (17)$$

These roots give the wave speed and damping of the various left- and right-running acoustic waves in the system. The first two roots $c_{1,2}$ represent modified-classical waves travelling at the isentropic wave speed and damped slightly by the presence of radiative transfer. Note that for these waves there is a separate contribution to the damping from each of the spectral bands. The other four roots represent the so-called radiation-induced waves [6, 7]. For this two-band model there are two sets of such waves, in contrast to the situation for a grey gas [7], where there is only one set. The additional set of radiation-induced waves is a direct consequence of the higher order of the present differential equation compared with that for a grey gas. To the present order of approximation, each set is due solely to the presence of one of the spectral bands. In this sense then, the wave speeds and dampings of these waves are uncoupled.

The extension of these approximate roots to include more bands is obvious: for each new band we add a damping term to the roots for the modified-classical wave, and write a new set of roots for a radiation-induced wave.

The amplitudes of the various waves are determined by the driving disturbance and boundary conditions. For the driving disturbance we assume that the waves are radiatively driven by a harmonic perturbation of the temperature of the wall at $\bar{x} = 0$ according to

$$\bar{T}_w = T_A e^{i\bar{t}}, \tag{18}$$

where T_A is the dimensionless amplitude of the driving disturbance.

Six boundary conditions are required. The first two of these are the fluid-dynamical conditions that the velocity must be zero at both walls, i.e.

$$\frac{\partial\bar{\varphi}}{\partial\bar{x}} = 0\bigg|_{\bar{x}\text{ and }\bar{L}=0} \tag{19}$$

where $\bar{L} \equiv L\omega/a_{S_0}$. At the driving wall two radiative boundary conditions are found by the procedure described in Section 2. One of these is, in terms of A_S and A_T,

$$\frac{d\bar{T}_w}{d\bar{t}} = \left\{ \frac{\left(1 + \dfrac{16a_1Bu_1/Bo_1}{16a_2Bu_2/Bo_2}\right)}{\left(1 - \dfrac{b_1^2Bu_1^2}{b_2^2Bu_2^2}\right)} \left(\frac{\dfrac{\partial^3 A_T}{\partial\bar{x}^3}}{b_2^3Bu_2^3} - \frac{\dfrac{\partial^2 A_T}{\partial\bar{x}^2}}{b_2^2Bu_2^2}\right) + \left(\frac{\dfrac{\partial A_T}{\partial\bar{x}}}{b_2Bu_2} - A_T\right) + \frac{Bo_2}{16a_2Bu_2}\left[\left(\frac{b_2^2Bu_2^2}{b_2^2Bu_2^2 - b_1^2Bu_1^2}\right)\left(\frac{\dfrac{\partial^4 A_S}{\partial\bar{x}^3\partial\bar{t}}}{b_2^3Bu_2^3} - \frac{\dfrac{\partial^3 A_S}{\partial\bar{x}^2\partial\bar{t}}}{b_2^2Bu_2^2}\right) + \left(\frac{b_1^2Bu_1^2}{b_1^2Bu_1^2 - b_2^2Bu_2^2}\right)\left(\frac{\dfrac{\partial^2 A_S}{\partial\bar{x}\partial\bar{t}}}{b_2Bu_2} - \frac{\partial A_S}{\partial\bar{t}}\right)\right]\right\}\Bigg|_{\bar{x}=0} \tag{20}$$

The other condition (not shown) is obtained simply by reversing the grey-band subscripts in this equation. At the reflecting wall and for harmonic waves the two radiative boundary conditions simplify to

$$\frac{\partial^3\bar{\varphi}}{\partial\bar{x}^3}\bigg|_{\bar{x}=\bar{L}} = \frac{\partial^5\bar{\varphi}}{\partial\bar{x}^5}\bigg|_{\bar{x}=\bar{L}} = 0. \tag{21}$$

In deriving the conditions (21) use has been made of the velocity boundary conditions (19). In none of the boundary conditions (20) and (21) however have we assumed that $16aBu/Bo \ll 1$. They could thus be used, if desired, to obtain a more general solution to the problem.

We now have sufficient conditions for a solution, since application of the six boundary conditions and equation (18) allows computation of the six constants C_m. The algebra is lengthy and will not be repeated here. The desired result is the pressure response at the reflecting wall, which we present for two representative conditions: (1) a nonresonant condition in which the distance between the walls is one-quarter of the classical acoustic wavelength ($\bar{L} = \pi/2$), and (2) a resonant condition in which the distance is one-half the classical acoustic wavelength ($\bar{L} = \pi$). In deriving these pressure-response equations we make continuing use, where necessary, of the approximation $16aBu/Bo \ll 1$; the results are thus applicable only in that approximation. With $\bar{p} \equiv p'/p_0$, the solutions are as follows: For the nonresonant case

$$\frac{\bar{p}(\bar{x} = \bar{L} = \pi/2)}{-iT_A e^{it}} = \frac{\dfrac{\gamma_0 b_1 Bu_1}{2}\dfrac{16a_1Bu_1}{Bo_1}[1 - \exp(-\pi b_1 Bu_1)]}{1 + b_1^2 Bu_1^2} + \frac{\dfrac{\gamma_0 b_2 Bu_2}{2}\dfrac{16a_2Bu_2}{Bo_2}[1 - \exp(-\pi b_2 Bu_2)]}{1 + b_2^2 Bu_2^2} + \frac{\gamma_0\dfrac{16a_1Bu_1}{Bo_1}\exp\left(-\dfrac{\pi}{2}b_1Bu_1\right)}{1 + b_1^2Bu_1^2} + \frac{\gamma_0\dfrac{16a_2Bu_2}{Bo_2}\exp\left(-\dfrac{\pi}{2}b_2Bu_2\right)}{1 + b_2^2Bu_2^2}, \tag{22}$$

and for the resonant case

$$\frac{\bar{p}(\bar{x} = \bar{L} = \pi)}{T_A e^{it}} = -\frac{\gamma_0}{\gamma_0 - 1}\left\{\frac{b_1Bu_1\dfrac{16a_1Bu_1}{Bo_1}}{(1 + b_1^2Bu_1^2)(1 + \coth \pi b_1Bu_1)} + \frac{b_2Bu_2\dfrac{16a_2Bu_2}{Bo_2}}{(1 + b_2^2Bu_2^2)(1 + \coth \pi b_2Bu_2)}\right\}$$
$$\times\left\{\frac{b_1Bu_1\dfrac{16a_1Bu_1}{Bo_1}}{1 + b_1^2Bu_1^2}\left[\frac{\pi}{2b_1Bu_1} + \frac{b_1^2Bu_1^2}{(1 + b_1^2Bu_1^2)(1 + \coth \pi b_1Bu_1)}\right] + \frac{b_2Bu_2\dfrac{16a_2Bu_2}{Bo_2}}{1 + b_2^2Bu_2^2}\left[\frac{\pi}{2b_2Bu_2} + \frac{b_2^2Bu_2^2}{(1 + b_2^2Bu_2^2)(1 + \coth \pi b_2Bu_2)}\right]\right\}^{-1}$$
$$- i\frac{\gamma_0\dfrac{16a_1Bu_1}{Bo_1}\exp(-\pi b_1Bu_1)}{(1 + b_1^2Bu_1^2)} - i\frac{\gamma_0\dfrac{16a_2Bu_2}{Bo_2}\exp(-\pi b_2Bu_2)}{(1 + b_2^2Bu_2^2)}. \tag{23}$$

For present purposes our principal interest in these solutions is to show from them the contributions and interactions (if any) of the two bands in producing the total response, so that we can generalize the solutions to multiple bands and thus improve the accuracy of spectral computations.

Consider first the response for the untuned condition [equation (22)]. Here the pressure is given by a sum containing four terms, all of which are of order either $1/Bo_1$ or $1/Bo_2$. The response from the single set of modified-classical waves is given by the first two terms, with each of these terms representing the contribution from one of the grey bands. The responses from the two sets of radiation-induced waves are given respectively by the last two terms, with each of these terms being due to only one band. Thus, for this untuned condition the pressure responses resulting from the two bands are uncoupled. If we had derived the one-grey-band equivalent of equation (22) we would have obtained only the first and third terms [8]. We can also recover the one-band solution directly from equation (22) either by setting one of the Bu's to zero or one of the Bo's to infinity (the latter corresponds to setting a bandwidth $\Delta\nu$ to zero) or by setting the Bu's equal and summing the $1/Bo$'s.

To generalize the untuned response for additional grey bands we need only add, for each of the additional bands, two more terms of the same form as those that already appear in equation (22). Following this approach we can extend the equation to an infinite number of grey bands each with spectral bandwidth $d\nu$, and thus obtain the response as an integral over spectral frequency. We accomplish this formally by first noting that as $\Delta\nu_j \to 0$, $1/Bo_j$ is proportional to $\Delta\nu_j$ [see equations (11) and (3)], and by then using the definition of an integral as the limit of a sum.* This procedure gives $\bar{p}$ as

$$\frac{\bar{p}(\bar{x} = \bar{L} = \pi/2)}{-iT_A e^{i\bar{t}}} = \frac{8\gamma_0}{Bo} \int_{\alpha_{\nu_0} \neq 0} \frac{a_\nu Bu_\nu}{(1 + b_\nu^2 Bu_\nu^2)} \frac{dB_\nu/dT|_0}{4\sigma T_0^3/\pi} \{b_\nu Bu_\nu[1 - \exp(-\pi b_\nu Bu_\nu)] + 2\exp[-(\pi/2) b_\nu Bu_\nu]\}\, d\nu. \tag{24}$$

In writing this equation we have redefined the Boltzmann and Bouguer numbers so that Bo is based on an infinite bandwith and Bu is based on the spectral absorption coefficient as follows:

$$Bo \equiv \frac{\rho_0 a_{S_0}^3}{(\gamma_0 - 1)\sigma T_0^4}, \qquad Bu_\nu \equiv \frac{\alpha_{\nu_0} a_{S_0}}{\omega}, \tag{25}$$

where σ is the Stefan–Boltzmann constant. The quantities a_ν and b_ν are exponential-approximation constants that (as discussed just prior to equation (2)) could be chosen, if desired, on the basis of the optical thickness of the gas at frequency ν.

Consider now the tuned response given by equation (23), which has a somewhat different form from that of equation (22). The first term—the ratio of the two factors in braces—is due to the modified-classical waves and has contributions from each band appearing in both numerator and denominator. Each of these contributions is of order either $1/Bo_1$ or $1/Bo_2$, so that as far as the dependence on Bo is concerned the response due to these waves is of order 1. Here, in contrast to the untuned situation, the way in which the two bands produce the response from the modified-classical waves is coupled; that is, the response is not simply a single sum of terms with one term from each band, but is the ratio of two sums. If we had derived the one-grey-band equivalent of this part of the response, we

* The damping of the modified-classical wave from equation (17) can also be written as an integral over spectral frequency by use of this technique. Such an expression might be useful for computing radiative damping of acoustic waves in planetary atmospheres (cf. Gille [9]) in which the assumption $16aBu/Bo \ll 1$ is usually well satisfied.

would have obtained a fraction that contained identical terms as in equation (23), but for only one band. This is exactly the same equation that we recover by specializing equation (23) to one band following the procedure mentioned earlier.

A physical interpretation of the tuned response from the modified-classical waves is helpful in justifying our subsequent extension of this response for multiple bands. In the numerator of that response the term from each band is proportional to the energy input to that band from the wall, while in the denominator the term from each band is proportional to the radiative damping, due to that band, of the modified-classical waves [see equation (17)]. Hence the contribution to the response from energy input into only one band is affected by the damping due to both bands. The complete tuned response from the modified-classical waves is the sum of such contributions.

The contribution of the radiation-induced waves to the tuned response is given by the last two terms in equation (23). This part of the response is similar to that in equation (22) for the untuned waves, both in algebraic form and in the fact that the contributions from the two bands are again uncoupled.

Despite the coupling of the bands in producing the response for the modified-classical waves, the generalization of equation (23) to include more bands is straightforward on the basis of the physical interpretation given above. For each new band we add terms, of the same form as already appear in equation (23), to the numerator and denominator of the fraction giving the response from the modified-classical waves, and to the sum of the terms giving the response from the radiation-induced waves. As before, we can further extend equation (23) to a continuous spectrum by generalizing to integrals over spectral frequency. Here we require three integrals, since there are three sums. In integral form the response becomes

$$\frac{\bar{p}(\bar{x} = \bar{L} = \pi)}{T_A e^{it}} = \frac{-\gamma_0}{\gamma_0 - 1} \frac{\displaystyle\int_{\alpha_{\nu_0} \neq 0} \frac{a_\nu b_\nu Bu_\nu^2}{(1 + b_\nu^2 Bu_\nu^2)(1 + \coth \pi b_\nu Bu_\nu)} \frac{\mathrm{d}B_\nu/\mathrm{d}T|_0}{4\sigma T_0^3/\pi} \mathrm{d}\nu}{\displaystyle\int_{\alpha_{\nu_0} \neq 0} \frac{a_\nu b_\nu Bu_\nu^2}{(1 + b_\nu^2 Bu_\nu^2)} \left[\frac{\pi}{2b_\nu Bu_\nu} + \frac{b_\nu^2 Bu_\nu^2}{(1 + b_\nu^2 Bu_\nu^2)(1 + \coth \pi b_\nu Bu_\nu)}\right] \frac{\mathrm{d}B_\nu/\mathrm{d}T|_0}{4\sigma T_0^3/\pi} \mathrm{d}\nu} - i\frac{16\gamma_0}{Bo} \int_{\alpha_{\nu_0} \neq 0} \frac{a_\nu Bu_\nu \exp(-\pi b_\nu Bu_\nu)}{(1 + b_\nu^2 Bu_\nu^2)} \frac{\mathrm{d}B_\nu/\mathrm{d}T|_0}{4\sigma T_0^3/\pi} \mathrm{d}\nu. \quad (26)$$

This equation is unusual in that it cannot be written as a single integral over spectral frequency, yet it represents a closed-form solution to a problem in radiative gas dynamics in which the details of the spectrum can be treated without approximation. We known of no way, other than the present generalization of the grey-band approach, by which this equation can be derived. It cannot, for example, be derived by the method of Cogley and Compton [5], because the terms neglected at the outset in their approximation are instrumental in producing equation (26).

Note that the Boltzmann number does not appear in the part of equation (26) resulting from the modified-classical waves. The resonant response is thus independent of *Bo* when these waves are dominant, which, because they are strongly enhanced due to resonance, occurs for all except very small values of *Bu*. This independence of *Bo* was also found numerically by Long and Vincenti [4] for a grey gas and large Boltzmann numbers.

We now discuss briefly the meaning and range of applicability of the approximation $16aBu/Bo \ll 1$. For simplicity, we confine ourselves to a grey gas, which in no way changes the essentials of the discussion. In terms of physical quantities and for a grey gas, $16aBu/Bo$ can be written as

$$\frac{16aBu}{Bo} = \frac{\dfrac{1}{\omega}\dfrac{4\pi a\alpha_0}{\rho_0}\displaystyle\int_0^\infty \left.\frac{dB_\nu}{dT}\right|_0 d\nu\, T'}{h'} = \frac{\dfrac{1}{\omega}\dfrac{4\pi a\alpha_0}{\rho_0}\left(\dfrac{4\sigma T_0^3}{\pi}\right)T'}{h'}. \tag{27}$$

The quantity in the numerator on the right-hand side of this equation is the perturbation in spontaneously emitted radiative energy in time a/ω per unit mass of gas; the quantity in the denominator is the perturbation in specific enthalpy. Hence, the present approximation states that the ratio of these two quantities must be small. At low and moderate temperatures this ratio is in fact small unless the absorption coefficient is very large or the acoustic frequency very low. The approximation therefore applies well in laboratory situations [8] and in planetary atmospheres. In stellar atmospheres, on the other hand, where temperatures are high, it would not be expected to apply well, if at all.

We now discuss briefly the relationship of the present work to that of Cogley and Compton [5]. The untuned response obtained here is essentially the same as that obtained in [5]. In fact, with the appropriate specializations and changes in nomenclature, the substitute-kernel counterpart of equation (41) of Cogley and Compton becomes identical with the monochromatic counterpart of the present equation (24). In contrast, the tuned responses are considerably different. The reasons for these facts reside in two important differences between the analyses, both of which affect the tuned, but not the untuned response.

The first of these is the presence of viscosity in the Cogley–Compton analysis. As mentioned at the beginning of Section 3, Cogley and Compton incorporated into the equations of motion a viscous term (to account for effects at the sidewall of a tube), which causes a viscous damping of the acoustic waves. (We could also have included that term in the present analysis, but we omitted it in order to emphasize the spectral aspects of the problem.) Because in the untuned situation the gas motion is small, the viscous damping has no influence (to the order of the solution) on the untuned response of [5]. Hence it is permissible to ignore the viscous aspect of the Cogley–Compton analysis in making the comparison between that work and the present work for the untuned situation. For the tuned situation, however, where gas motion is relatively large, the presence of the viscous damping exerts an important influence and leads to the differing results.

The second difference between the analyses is in the way the assumption $16aBu/Bo \ll 1$ has been applied. Cogley and Compton in effect made the assumption at the outset that the temperature perturbation in the gas relative to the temperature perturbation of the wall, T'/T'_w, was a small quantity of order $16aBu/Bo$ and, consistent with this assumption, retained terms of order $16aBu/Bo$ in their analysis. The present work makes the less restrictive assumption that $16aBu/Bo \ll 1$, which places no restriction on the size of the temperature perturbation [other than that it is small enough to satisfy the small-disturbance equations (1) and (9)]. For the untuned situation, the temperature perturbation computed on the basis of the present approximation is indeed found to be of order

$16aBu/Bo$; hence the agreement of equation (24) with the Cogley–Compton analysis. For the present tuned situation, with the only damping being radiative, T'/T'_w is found to be of order unity, thus violating Cogley and Compton's basic assumption. In fact, their method, when applied to the present inviscid problem, predicts an infinite tuned response. They were able in their work to obtain an expression for a tuned response only because they included the side-wall viscous damping. As noted in their paper, their tuned result is valid only when the viscous damping is much larger than the radiative damping of the modified-classical wave; and the converse, obviously, is true of the present, inviscid, tuned result.

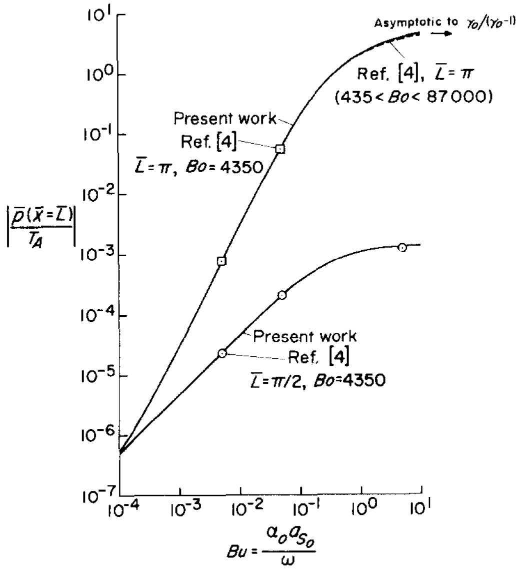

FIG. 1. Dimensionless pressure amplitude at reflecting wall as a function of *Bu* for a grey gas with $\gamma_0 = 1{\cdot}25$.

Finally, in Fig. 1 we compare the present results, specialized to a grey gas, with the results given by Long and Vincenti. Those authors did not make the assumption that $16aBu/Bo \ll 1$, but did a complete numerical solution (computer-generated) to the full linearized grey-gas equations. In making this comparison we have chosen $a = 1$ and $b = \sqrt{3}$ to correspond (approximately) to Long and Vincenti's use of the differential approximation. The agreement is seen to be very good. The small differences result primarily from a slight difference between the radiative boundary conditions derived for the differential approximation and those derived for the exponential approximation with $a = 1$ and $b = \sqrt{3}$. This comparison in effect verifies the validity of the approximation $16aBu/Bo \ll 1$ for the range of parameters shown on Fig. 1.

5. CONCLUDING REMARKS

The development of the differential equation (8) for the multiple-band radiative heat flux is dependent here on both the one-dimensional geometry and linearization of the radiative-flux equation. These specializations are appropriate for the application to one-dimensional radiating acoustic flow. Neither, however, is critical. Equation (8) can be generalized both for multi-dimensional geometries within the framework of the differential approximation, and also for a nonlinear situation. References [2] (multi-dimensional, nonlinear) and [3] (one-dimensional, nonlinear) provide two-grey-band examples. The application of this technique is thus not restricted to problems of radiative acoustics.

ACKNOWLEDGEMENTS

This work was the outgrowth of a research program supported by the Office of Naval Research under Contract N00014-67-A-0112-0017, and by the U.S. Air Force Office of Scientific Research under Contract F44620-69-C-0006. The author's time at Stanford was supported by a gradute-study program of the Ames Research Center, National Aeronautics and Space Administration.

The author gratefully acknowledges helpful discussions of the subject matter with Gary T. Chapman and with Professors Donald Baganoff and Walter G. Vincenti.

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FORMULATION À BANDES MULTIPLES POUR LE TRANSFERT PAR RAYONNEMENT DANS UN GAZ LÉGÈREMENT TROUBLÉ, AVEC APPLICATION À DES ONDES ACOUSTIQUES SE PROPAGEANT DE FAÇON RAYONNANTE

Résumé—Une équation différentielle linéarisée relative à un flux de chaleur rayonnant est obtenue pour un gaz non gris qui est en équilibre moléculaire local et qui est légèrement perturbé autour de son équilibre rayonnant. Cette équation est établie pour une géométrie à plans parallèles en supposant que le coefficient d'absorption spectral peut être représenté par un nombre arbitraire de bandes grises et en utilisant l'approximation (exponentielle) apprópriée. L'ordre de l'équation est 2 N où N représente le nombre de bandes grises. On discute la formulation des conditions aux limites associées à l'équation. L'équation est applicable aux problèmes de transfert par rayonnement monodimensionnel chaque fois que l'on considère des petites perturbations à partir d'un état de référence uniforme.

L'intérêt de l'équation est démontré en l'utilisant pour résoudre un problème d'acoustique, spécifiquement celui d'ondes acoustiques harmoniques conduites de façon rayonnante dans un gaz entre deux parois. Des solutions analytiques concernant deux bandes grises sont obtenues pour la réponse de pression à la paroi non émettrice sur laquelle existe une approximation de température basse ou modérée. Ces solutions, relatives aux deux bandes grises sont telles que l'on peut les étendre d'abord à de multiples bandes grises par les sommations adequates, et finalement à un spectre continu par un passage à des intégrales sur la fréquence spectrale. Cette extension finale, en effet, modifie l'approximation originale du modèle de bandes dans ce problème d'acoustique particulier.

EIN VIELBANDENANSATZ FÜR DEN STRAHLUNGSAUSTAUSCH IN EINEM LEICHT GESTÖRTEN GAS, MIT ANWENDUNG AUF STRAHLUNGSBEDINGTE AKUSTISCHE WELLEN

Zusammenfassung—Eine linearisierte Differentialgleichung wird angegeben für den Wärmefluss durch Strahlung im Falle eines nicht-grauen Gases, das in lokalem molekularem Gleichgewicht steht und vom Strahlungsgleichgewicht her leicht gestört ist. Diese Gleichung wurde für eine plan-parallele Geometrie abgebildet, wobei für die spektralen Absorptionskoeffizienten eine willkürliche Anzahl von grauen Banden angenommen und die (exponentielle) "Ersatzkernnäherung" verwendet wurde. Die Gleichung ist von 2 N-ter Ordnung, wobei N gleich der Anzahl der grauen Banden ist. Es wird auch das Aufstellen von Randbedingungen zum Gebrauch der Gleichungen diskutiert. Die Gleichung ist bei allen Problemen eindimensionaler Wärmestrahlung anwendbar, falls klein Abweichungen von einem homogenen Bezugszustand betrachtet werden.

Die Brauchbarkeit der Gleichung wird dadurch gezeigt, dass mit ihrer Hilfe ein Problem der "Strahlungsakustik"—speziell das der durch Strahlung verursachten harmonischen akustischen Wellen in einem Gas zwischen zwei Wänden—gelöst wird. Analytisch werden Zwei-Graubanden-Lösungen für das Drucksignal an der passiven Wand abgeleitet, was, praktisch eine Näherung für niedrige oder mittlere Temperaturen darstellt. Diese Zwei-Graubanden-Lösungen sind so geartet, dass sie zunächst durch passende Summationen zu Viel-Graubanden Lösungen erweitert und schliesslich auf ein kontinuierliches Spektrum durch den Übergang zu Intergralen über die spektralen Frequenzen verallgemeinert werden können. Diese letzte Verallgemeinerung beseitigt in diesem speziellen akustischen Problem die ursprüngliche Banden-Modell-Näherung.

МНОГОПОЛОСНАЯ ТРАКТОВКА ЛУЧИСТОГО ПЕРЕНОСА В СЛАБО ВОЗМУЩЕННОМ ГАЗЕ В ПРИЛОЖЕНИИ К ГЕНЕРИРУЕМЫМ ИЗЛУЧЕНИЕМ АКУСТИЧЕСКИМ ВОЛНАМ

Аннотация—Получено линеаризованное сугубо дифференциальное уравнение лучистого теплового потока для несерого газа, находящегося в локальном молекулярном равновесии и слегка отклоняющегося от лучистого равновесия. Это уравнение выведено для плоскопараллельной геометрии в предположении возможности выражения коэффициента спектрального поглощения через произвольное число серых полос и с помощью (экспоненциальной) аппроксимации путем замены ядра. Порядок уравнения равен $2N$, где N есть число серых полос. Рассмотрены также граничные условия данного уравнения. Уравнение можно использовать в одномерных задачах лучистого переноса при рассмотрении небольших отклонений от однородного начального состояния.

Применимость уравнения демонстрируется на примере решения задачи лучистой акустики, а именно, на примере генерируемых излучением акустических волн в газе между двумя стенками. Решения с двумя серыми полосами получены для изменения давления на неподвижной стенке с помощью аппроксимации низких или средних температур. Эти решения с двумя серыми полосами представляют собой такие решения, которые путем выполнения соответствующих суммирований сначала можно применить к многочисленным серым полосам и, наконец, к бесконечному спектру с помощью перехода к интегралам по спектральной частоте. Фактически последнее исключает первоначальную аппроксимацию с помощью модели полос в этой частной акустической задаче.